Labor Market Volatility in the RBC Search Model: A Look at Hagedorn and Manovskii's Calibration[*]

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Abstract

The standard Diamond-Mortensen-Pissarides (DMP) labor search model generates low volatility in labor market variables relative to average labor productivity (ALP), the so-called Shimer puzzle. Hagedorn and Manovskii (2008) demonstrate that recalibrating the standard DMP model to be consistent with the small vacancy posting cost and wage elasticity observed in the data can resolve the Shimer puzzle. They close by stating that their calibration strategy would also resolve the Shimer puzzle in the real business cycle (RBC) search framework. In this paper, we examine their claim and find that their strategy resolves the Shimer puzzle in the RBC search model for linear preferences (with risk neutrality and infinite Frisch elasticity of labor supply), but falls significantly short for more standard assumptions on the degree of relative risk aversion (of 1-2) and Frisch elasticity (of 2-3), in line with empirical estimates. While our conclusions are based on highly accurate solutions using the Generalized Stochastic Simulation Algorithm (GSSA), we also assess the accuracy of a frequently used linearization method and its implications for the assessment of labor market volatility.

Keywords: GSSA, Labor Market Search, Shimer Puzzle, Real Business Cycle Model

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1   Introduction

The Shimer puzzle, or the observation that the standard Diamond, Mortensen, and Pissarides (DMP) labor market search model fails to match the high volatility of labor market variables (vacancies, unemployment, and their ratio, labor market tightness) relative to average labor productivity (ALP) that is observed in the U.S. data is well known.[1] Shimer (2005) notes that in the DMP search model, wages tend to absorb fluctuations in productivity leading to small adjustments in vacancies and unemployment over the business cycle. Real Business Cycle (RBC) search models (RBC models with DMP search friction) inherit this lack of labor market volatility (see Andolfatto, 1996; Merz, 1995; Shimer, 2005; and Hagedorn and Manovskii, 2008).

Shimer (2005), Hall (2005), Hall and Milgrom (2008), and Pissarides (2009) show that introducing wage stickiness in the standard DMP framework can resolve the Shimer puzzle.[2] Hagedorn and Manovskii (2008) (HM) propose an alternative solution that simply involves recalibrating the DMP model. We show that while the HM calibration resolves the Shimer puzzle in the extended RBC search model when preferences are linear, as they are in the standard DMP model, this is not the case once agents are risk averse. In addition, there is a further deterioration of the model's predictions when the agents' Frisch elasticity of labor supply is made finite. These findings suggest that the success of the HM calibration was driven, in part, by the assumption of linear preferences embedded in the DMP model. Given the non-linearities (in the utility function and production function) inherent in the modifications of the standard DMP model needed to place it in the RBC framework, we approximate a solution to our model nonlinearly. Specifically, we employ the Generalized Stochastic Simulation Algorithm (GSSA) described by Judd et al (2011) and Maliar and Maliar (2014) to obtain a nonlinear approximation, and we demonstrate that this approximation yields a highly accurate solution, according to the DM-statistics test (see den Haan and Marcet, 1994).

Prior to HM, a standard calibration of the DMP model would set worker's bargaining power to either yield a symmetric Nash bargain or to satisfy the Hosios criteria for the

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[1]See Diamond (1982), Mortensen and Pissarides (1994) and Pissarides (1985) for details of the DMP model and Shimer (2005) for a discussion of the Shimer puzzle.

[2]Hall and Milgrom (2008) introduce periodic renegotiation costs into the wage bargaining problem to calibrate the degree of wage stickiness required to match the data. Pissarides (2009) offers an alternative approach by suggesting that Nash bargaining on new hires only (which he finds to be consistent with the data) along with the addition of a fixed cost to hiring can match both the wage elasticity and the labor market tightness elasticity with respect to productivity.

Pareto optimality of the decentralized solution, and set the disutility of work to target the replacement ratio observed in the data. With worker's bargaining power and disutility of work pinned down, vacancy posting costs and the matching function parameter would be set to target the average values for the job finding rate and labor market tightness that are found in the data.[3] HM disagree with this procedure as it ignores the size of vacancy posting costs and the elasticity of wages with respect to average labor productivity, which are both found to be very small in the data.

HM state that the primary issue with the standard calibration is its failure to match the small vacancy posting costs observed in the data. In DMP models, firms pay workers less than their marginal products in order to recoup their vacancy posting costs. As such, firms earn period-by-period accounting profits whose size is determined by the size of the vacancy posting costs present in the model. The central idea behind the HM calibration is that these profits should be small and they should fluctuate significantly in percentage terms at business cycle frequencies. Large percentage fluctuations in firm's accounting profits give rise to strong incentives to adjust vacancy creation, leading to large movements in unemployment and labor market tightness. In order to capture the small and volatile accounting profits, HM sets vacancy posting costs to a small level that is consistent with the data, and they set worker's bargaining power to target the elasticity of wages with respect to average labor productivity. With these two values pinned down, the disutility of work and the matching function parameter are set to target the average value of the job finding rate and labor market tightness found in the data. This calibration strategy yields a value for the replacement ratio of 0.96.[4].

The HM calibration succeeds in increasing the relative volatility of labor market variables to realistic levels in the DMP model. However, the DMP model is special – it assumes risk neutral agents, constant disutility of labor (or equivalently, an infinite Frisch elasticity), and a production process that is linear in labor. HM close their paper by stating that their calibration strategy will also resolve the Shimer puzzle in the more general RBC search framework, where agents are typically risk averse and the production process includes capital as an additional input. In this paper, we put this claim to the test by implementing the HM calibration strategy in a fully specified RBC search model using a standard Cobb-Douglas

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[3]This calibration procedure is very similar to that outlined in Shimer (2005).

[4]Many researchers question the plausibility of such a high replacement ratio (See Hornstein et al (2005), Mortensen and Nagypal (2007), and Costain and Reiter (2008)). For this paper we will assume that a replacement rate close to 1, implied by HM calibration, is acceptable and focus on the performance of their calibration within the RBC search model.

production function and a variety of commonly used utility specifications. In order to ensure an accurate approximation, we use the Generalized Stochastic Simulation Algorithm (GSSA) to solve our model nonlinearly. However, as there is currently an open debate regarding the importance of numerical accuracy in models with labor market search frictions (see Petrosky-Nadeau and Zhang, 2016 and Lan, 2017), we also solve a linear approximation to our model. While we find that this linearized solution fails the accuracy test based on DM-statistics, the basic conclusions are consistent across solution methods.

We find that while the HM calibration significantly amplifies the volatility of labor market variables relative to average labor productivity (ALP) for the various utility specifications, it is able to match the data and resolve the Shimer puzzle only when one assumes that agents are risk neutral. Increasing the degree of relative risk aversion to unity (which is at the lower end of the range used in most RBC studies), sharply reduces labor market volatility under the HM calibration. The heightened consumption-smoothing behavior reduces the intertemporal elasticity of substitution in consumption and renders the consumption-savings decision less responsive to productivity shocks thereby mitigating the demand for labor by firms. Maintaining a low wage elasticity with respect to average labor productivity coincides with a weaker employment response. Thus, our results indicate that the success of the HM calibration in resolving the Shimer puzzle is at least partially dependent on the assumption of risk neutrality embedded in the standard DMP model. A similar effect results when the Frisch elasticity is lowered to values generally in accord with macro estimates. These results suggest that if one is interested in resolving the Shimer puzzle within an RBC search model under conventional values of risk aversion and Frisch elasticity, the HM calibration may get you closer to the data, but additional amplification mechanisms, such as financial frictions, are needed. (See Atolia, Gibson, Marquis, 2015). Lastly, we find that while the GSSA approximation is much more accurate than the linearized solution in terms of Euler residuals and DM-statistics, the main conclusions derived from the model remain unchanged.

2   Model

The model is a standard Real Business Cycle (RBC) model that has been modified to include a basic labor market search friction. This structure is similar to that presented in Merz (1995) and Andolfatto (1996), but some details have been adjusted to maintain a tight connection with Hagedorn and Manovskii (2008) (HM).

2.1 Labor Market

There is a continuum of workers of unit mass within the household that are either employed, n, or unemployed, $1 - n$. Workers are insulated from the consumption risk associated with unemployment spells through their membership in this multi-member household. Employed workers supply labor to firms at the bargained wage, w, while unemployed workers search for work. Firms post vacancies, v, at flow cost g each period in order to expand their workforce.

New matches, m, are determined by the following matching function:

$$m = \frac{v(1-n)}{((1-n)^\gamma + v^\gamma)^{\frac{1}{\gamma}}}, \quad \gamma \in (0,1) \tag{1}$$

where this functional form has been chosen to retain comparability with HM.

Labor market tightness, Φ, the job finding rate, f, and the vacancy filling rate, q, are given by:

$$\Phi = \frac{v}{1-n} \tag{2}$$

$$f = \frac{m}{1-n} \tag{3}$$

$$q = \frac{m}{v} \tag{4}$$

Separations occur exogenously at rate x, so that the evolution of labor is given by:

$$n' = (1-x)n + m, \quad x \in (0,1) \tag{5}$$

2.2 Household's Problem

Households derive utility from consumption, c, and disutility from work, n. They choose consumption and next-period capital, k', to maximize the present discounted value of lifetime utility, taking the wage rate, w, the rental rate on capital, r, and the job finding rate, f, as given. The household's problem can be stated as the following dynamic program:

$$V(k,n;\theta) = \max_{c,k'} \{U(c,n) + \beta E\left[V(k',n';\theta')\right]\} \tag{6}$$

s.t.

$$c + k' - (1-\delta)k \le wn + rk \tag{7}$$

$$n' = (1-x)n + f(1-n) \tag{8}$$

where β denotes the household's discount factor, θ denotes an exogenous aggregate productivity shock, equation (7) is the household's budget constraint, and equation (8) governs the evolution of employment from the point of view of the household.

Solving the household's problem yields the following dynamic equations:

$$\beta E \left[\frac{U_{c'}}{U_c}[r' + 1 - \delta] \right] = 1 \tag{9}$$

$$V_n = U_c w + U_n + \beta(1 - x - f)E\left[V_{n'}\right] \tag{10}$$

Equation (9) is the standard capital Euler equation and equation (10) is the marginal value of an additional employed worker to the household.

2.3 Firm's Problem

The firm combines capital, k, and labor, n, to produce output, y, using the following technology:

$$y = \theta k^\alpha n^{1-\alpha} \tag{11}$$

where the exogenous aggregate productivity shock, θ, evolves according to:

$$\ln \theta' = \rho_\theta \ln \theta + \epsilon'_\theta \tag{12}$$

with $\epsilon'_\theta \sim N(0, \sigma_\epsilon^2)$ and $\rho_\theta \in (0,1)$.

Firms rent capital and post vacancies to maximize the present discounted value of lifetime profits based on household preferences taking the wage rate, w, the rental rate on capital, r, the cost of posting a vacancy, g, and the vacancy filling rate, q, as given. Their problem can be stated as the following dynamic program:

$$J(n; \theta) = \max_{k,v} \left\{ \theta k^\alpha n^{1-\alpha} - wn - rk - vg + \beta E \left[\frac{U_{c'}}{U_c} J(n'; \theta') \right] \right\} \tag{13}$$

$$n' = (1 - x)n + qv \tag{14}$$

where equation (14) is the evolution of employment from the point of view of the firm.

Solving the firm's problem yields the following optimality conditions:

$$r = \alpha\theta\left(\frac{k}{n}\right)^{\alpha-1} \tag{15}$$

$$\frac{g}{q} = \beta E\left[\frac{U_{c'}}{U_c}J_{n'}\right] \tag{16}$$

$$J_n = (1-\alpha)\theta\left(\frac{k}{n}\right)^{\alpha} - w + (1-x)\frac{g}{q} \tag{17}$$

Equation (15) determines the rental rate, equation (16) is the firm's vacancy creation condition, and equation (17) is the marginal value of an additional worker to the firm.

2.4 Wage Bargain

Wages are set through Nash bargaining over the total surplus generated by a successful match. Typically, one defines total surplus, S, as the value of an additional employed worker to the household, V_n, plus the value of an additional employee to the firm, J_n. However in our case, V_n is in terms of utils while J_n is in terms of good. To deal with this issue simply define $\bar{J}_n = U_c J_n$ and $S = V_n + \bar{J}_n$. Now, wages are set by solving the following problem:

$$\max_{V_n, \bar{J}_n} V_n^b \bar{J}_n^{1-b}$$

s.t.

$$S = V_n + \bar{J}_n$$

where b is the worker's bargaining power.

Solving this problem yields:

$$(1-b)V_n = bJ_n U_c \tag{18}$$

One can use equations (10), (17), and (18) to derive a reduced-form wage equation:

$$w = -\frac{U_n}{U_c} + b\left((1-\alpha)\theta\left(\frac{k}{n}\right)^{\alpha} + \Phi g + \frac{U_n}{U_c}\right) \tag{19}$$

2.5 Market Clearing and Equilibrium Conditions

The goods market clearing condition is given by:

$$c + k' - (1-\delta)k + vg = \theta k^{\alpha} n^{1-\alpha} \tag{20}$$

Once one derives equation (19) and substitutes an updated version of equation (17) into equation (16), J_n and V_n are redundant to the system. Therefore, the model consists of 10 endogenous variables: m, v, n, Φ, f, q, c, k, w, r and the system of equations that define the model consists of equations (1)-(5), (9), (15), (16), (19), and (20).

3   Calibration and Computational Methods

This section specifies the utility function that we consider and outlines our calibration method, which closely mimics that of Hagedorn and Manovksii (2008). This is followed by a discussion of the methods used to numerically solve the model.

3.1   Utility Function

We restrict our attention to preferences with the following general form:

$$U(c,n) = \frac{1}{1-\sigma}c^{1-\sigma} - \eta\frac{1}{1+\frac{1}{\xi}}n^{1+\frac{1}{\xi}}, \quad \sigma \geq 0, \xi > 0 \tag{21}$$

where σ is the coefficient of relative risk aversion (and the inverse of the elasticity of intertemporal substitution), η is the measure of disutility of work, and ξ is the Frisch elasticity of labor supply. This is a standard utility specification used in the RBC literature and subsumes many specifications used in labor market search models, for example, those used in Merz (1995) and Shimer (2010).

Initially, we set $\sigma = 0$ and $\xi = \infty$, which is equivalent to the linear utility function assumed by HM. We find that the HM calibration still resolves the Shimer puzzle under these utility parameters. However, these parameter values are highly restrictive as they imply risk neutral agents who have infinite Frisch elasticity. A more reasonable parameterization, from an RBC perspective, would allow for plausible values of risk aversion and (finite) Frisch elasticity. We resolve the model under a variety of utility parameterizations and find that the HM calibration's ability to resolve the Shimer puzzle depends crucially on the specific values of utility parameters σ and ξ that are chosen.

3.2   Calibration Procedure

In order to maintain consistency with HM, we set our model's period length to $\frac{1}{12}$ of a quarter, or approximately one week. Following the RBC literature, we set the discount rate,

β, to $0.99^{\frac{1}{12}}$, which implies an annual risk free rate of approximately 4%. Also following convention, we set capital's share of output, α, equal to $\frac{1}{3}$.[5] As for the depreciation rate of capital, we set $\delta = \frac{0.025}{12}$, implying an annual depreciation rate of 10%. This value of δ allows our model to match our target capital-output ratio of 2.20 which is consistent with the U.S. economy over the sample period 1970-2015.[6]

To ensure comparability with the HM calibration, the exogenous separation rate, x, is set to 0.0081. For the same reason, the utility parameters, σ and ξ are initially set to 0 and ∞, although other combinations of σ and ξ are also used for sensitivity analysis. At this point we have values for σ, ξ, β, δ, x, and α, but we still need values for g, σ_ϵ, ρ_θ, γ, η, and b. Following HM, vacancy posting costs at time t are given by:

$$g_t = 0.474 \left(\frac{y_t}{n_t} \right) + 0.11 \left(\frac{y_t}{n_t} \right)^{0.449} \tag{22}$$

where the coefficients represent the estimates of the capital and labor share of vacancy costs respectively, and the exponent is added to capture the sluggishness in the response of wages to ALP.[7] With the equation for g specified, we proceed to set the values for the five remaining parameters (σ_ϵ, ρ_θ, γ, η, and b) so that our model matches the five remaining targets proposed by HM. These five targets are: (i) volatility of ALP of 1.3%, (ii) first-order autocorrelation of ALP of 0.765, (iii) average labor market tightness of 0.634, (iv) average job finding rate of 0.139, and (v) the elasticity of wages with respect to ALP of 0.449.[8] Note that this part of the calibration is repeated for each of the different combinations of σ and ξ reported in Tables 1 and 2. The two tables provide a complete list of parameter values for the different (σ, ξ) combinations that we use. Note that the parameter values are nearly identical between specifications and across the two alternative approximation methods that we wish to compare. The next section describes these two approximation methods.

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[5]In RBC search models that employ a Cobb-Douglas production function, α is still capital's share of output, but $1 - \alpha$ is no longer labor's share. This is because vacancy costs come out of labor's share. However, it is still close to $1 - \alpha$ because under the HM calibration, vacancy costs are very small—of the order of 1 to 2 percent of GDP.

[6]The capital-output ratio is defined as the ratio of Current Cost Net Stock of Private Fixed Assets to GDP as in Evans (2000) and, although we also include more recent data, our value is almost the same as what he reports in his Figure 2.

[7]The inclusion of capital in our model does not alter the vacancy posting costs described in HM. However, ALP is no longer normalized to 1.

[8]These targets are taken directly from HM and can be found in Tables 1 and 3 of their paper.

3.3 Computational Methods

It is customary in the literature to solve labor market search models by using (log) linearization. However, as discussed in more detail in the next section, recent work has suggested that the model-implied second moments obtained from labor search models may be highly sensitive to the accuracy of approximation. In light of this, we use the highly accurate Generalized Stochastic Simulation Algorithm (GSSA) proposed by Judd et al (2011) and Maliar and Maliar (2014) to solve our model. A third-order approximate solution computed using GSSA passes the DM-statistics test, based on Euler equation residuals, as proposed by den Haan and Marcet (1994). However, with a view of providing further evidence to inform the debate on the importance of accurately approximating labor search models, we also solve a linear approximation of our model using the method of undetermined coefficients outlined in Christiano (2002) and assess its accuracy using the DM-statistics test.

3.3.1 Generalized Stochastic Simulation Algorithm

GSSA is a generalization of the parameterized expectations algorithm (PEA) proposed by den Haan and Marcet (1990) where the approximating function is chosen from a family of orthogonal polynomials and accurate quadrature techniques replace single-node Monte Carlo integration when evaluating conditional expectations.[9] In principle, both GSSA and PEA can be used to approximate the solution to our model with an arbitrary accuracy: They do not suffer from the limitations of standard techniques, such as linearization. However, GSSA is a significant improvement over PEA, as the use of multi-node quadrature integration versus single-node Monte Carlo integration when evaluating conditional expectations allows GSSA to achieve a similar degree of accuracy with a much shorter simulation path. In fact, *sans* GSSA it would have been hardly possible to accurately solve our model at a weekly frequency to check the performance of the HM calibration.

For our approximation, we use a third-order Hermite polynomial in the model's state variables to approximate the expectation functions and we use 9-node Gauss-Hermite quadrature to evaluate integrals. In order to pass the DM-statistics test and achieve an acceptable level of accuracy (as discussed later), we simulate the model for 120,000 periods. While 120,000 periods seems large, this is actually a significant reduction from what would be needed with

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[9]In traditional PEA conditional expectations are evaluated using the next period's realized shock. This is equivalent to performing Monte Carlo integration with a single integration node. GSSA replaces this by evaluating expectations numerically using fixed quadrature techniques. This allows GSSA to achieve a significantly higher degree of accuracy with fewer simulation periods.

PEA.[10] A Technical Appendix that outlines our computational strategy is available from the authors upon request.

4 Results

In this section, we present our results in steps. First, we assess the ability of the HM calibration to resolve the Shimer puzzle in an RBC search model with linear preferences. This initial specification replicates the utility function assumed by HM. Second, we test the robustness of our findings to changes in utility parameters. Specifically, we consider alternative preference specifications that are commonly found in the RBC literature and allow for risk-aversion (σ significantly greater than zero) and increasing marginal disutility of labor supply (finite Frisch elasticity). In view of the sensitivity of the results to the accuracy of the solution methods used, as highlighted in the recent literature, our discussion of the Shimer puzzle relies on highly accurate results obtained using GSSA. Third, to further inform the debate on the importance of the accuracy of solution methods, we also present results from a linear approximation of the model. We formally test the accuracy of both solution methods and assess the impact of inaccuracies introduced by linearization.

4.1 Resolving the Shimer Puzzle: Linear Preferences as in HM

We start by approximating the solution to our model for linear preferences ($\sigma = 0$ and $\xi = \infty$) which are equivalent to the preference specification employed by HM. The top panel of Table 3 presents the empirical targets that need to be met in order to implement the HM calibration. As for the empirical fit of this specification, our model, approximated using GSSA, returns values of 15.0, 12.4, and 25.3 for the (percent) volatilities of vacancies, unemployment and labor market tightness, which are extremely close to the data reported by HM: 13.9, 12.5, and 25.9. In fact, our results in this RBC framework are actually closer to the data than the model results reported by HM.

Before discussing the Shimer puzzle further, we must assess the accuracy of our GSSA solution under our linear preference specification. Results from our accuracy tests can be found in the column labeled "Linear" in Table 4. The table reports Euler equation residuals in the first panel and results from a DM-statistics test on the predictability of ex-post errors

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[10]We find that even after doubling the simulation length, PEA is unable to come close to the accuracy of our GSSA approximation. Thus, for our model, using PEA to obtain an accurate approximation is prohibitively expensive in terms of computer time.

in the Euler equations in the second. While our model has two Euler equations, (9) and (16), we are able to solve equation (9) analytically given the log-normal distribution of θ when preferences are linear. For this reason, we only have to report the Euler residuals and DM statistics tests for equation (16) for this preference specification.[11] The residuals for equation (16) are calculated from the time paths of model variables computed using solved decision rules and a 9-node Gauss-Hermite quadrature approximation of the conditional expectations. The first panel of Table 4 presents both the root-mean-squared error (MSE) and the maximum-absolute error (∞-norm) for this residual series. The DM-statistics tests in the second panel are computed following den Haan and Marcet (1994). For computing DM-statistics, the solved decision rules are used to simulate 1000 instances of 3100-period time paths of the model's variables, with the first 100 periods being used for "burn in."[12] The values reported in the second panel of Table 4 measure the fraction of DM-statistics that fall above (below) the 95% (5%) critical value. We see that, for linear preferences, (16) shows small approximation errors. More generally, we note that the GSSA solution passes the DM-statistics test with approximately 5% of DM-statistics falling within the appropriate tails uniformly across various specifications considered in Table 4.

In addition to the GSSA solution, which gets very close to data and is highly accurate as discussed above, in Table 3 we also present results from a frequently used linear approximation. The linearized solution generates significantly higher volatility of labor market variables, but as expected, fails our DM-statistics test (see first column of Table 6). Specifically, the volatilities increase to 20.16, 17.48, and 34.35 respectively. The results of HM lie somewhere in between those from the highly accurate GSSA solution and the commonly used linear approximation. The overall conclusion, therefore, is that the HM calibration successfully resolves the Shimer puzzle in an RBC search model when agents are assumed to be risk neutral and to have Frisch elasticity of infinity (i.e., linear preferences).

These results indicate that the HM calibration has the potential to resolve the Shimer puzzle in a standard RBC search model; however, further analysis is needed. The reason is the very stark assumption about the preferences on which these results are based. The general consensus in the macro literature is to put much more curvature in preferences, which implies some/higher (relative) risk-aversion and lower Frisch elasticity in our utility

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[11]For all other preference specifications, we will have to consider the residuals for both (9) and (16). Furthermore, this means that the correct DM statistics test will be a joint test of both equations.

[12]When computing the DM-statistics, we use lagged consumption and lagged productivity as our independent variables in the regression step. Therefore, our DM-statistics are distributed χ^2 with 2 degrees of freedom.

specification. For example, Jung and Kuester (2011) have demonstrated that the volatility of labor market variables falls as the degree of relative risk aversion rises, suggesting that once we move to a more realistic utility specification, the HM calibration may fail to resolve the Shimer puzzle. However, while Jung and Kuester (2011) follow HM and set workers' bargaining power to target the elasticity of wages with respect to ALP, they do not implement the HM calibration strategy in totality. Therefore, their findings do not conclusively demonstrate if, and the extent to which, increasing risk aversion reduces labor market volatility in an RBC search model under the HM calibration.[13]

4.2 Resolving the Shimer Puzzle: RBC Preferences

Our previous results indicate that the HM calibration is sufficient to resolve the Shimer puzzle in an RBC search model with linear preferences. However, this specification is not consistent with the RBC literature, which typically assumes curvature in preferences. To assess how increasing curvature (in accordance with RBC/ macro literature) affects the HM calibration's ability to resolve the Shimer puzzle, we solve our model under two alternative values for the coefficient of relative risk aversion: $\sigma = 1$ and $\sigma = 2$. These values of σ are widely used in the RBC literature, with $\sigma = 1$ (implying log preferences) being used frequently in analytical work, and $\sigma = 2$ being used more often in quantitative exercises. To further test the effects of additional curvature in preferences, we also consider alterative values for the Frisch elasticity that are more in accord with the macroeconomic estimates in the range of 2-3.[14] To this end, we solve our model for values of the Frisch elasticity equal to ∞, 3, and 2 for each of the new values of σ considered, implying a total of six alternative preference specifications.

Table 4 reports the accuracy of our GSSA approximation for each the six alternative RBC preference specifications. As mentioned earlier, all specifications pass accuracy test

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[13]We note here that, in a well-executed paper, Lubik (2009) uses Bayesian techniques to estimate an RBC search model with CES utility after imposing the restriction of infinite Frisch elasticity. He obtains very low point estimates for both the degree of relative risk aversion (0.72) and worker's bargaining power (0.03). While these estimates are consistent with HM, Lubik (2009) includes several additional shocks (to demand, labor supply, and matching efficiency) for estimation, with shocks to match efficiency serving as the primary driver of labor market volatility. Therefore, his results do not of themselves offer a theoretical explanation for resolving the Shimer puzzle, which was the objective of HM and the focus of this paper.

[14]In their survey of the literature, Reichling and Whalen (2012) suggest a range of 2 to 4. Prominent examples include Hall (2009), whose calibration exercise produced a macro Frisch elasticity of labor supply in a sticky-wage model of 1.9 and Rogerson and Wallenius (2009), whose calibration exercises in an OLG model with labor taxation produced estimates ranging from 2.3 to 3.0. Smets and Wouters (2007) employ parameter estimation in a DSGE model to obtain a point estimate of 1.9.

based on DM-statistics. With the accuracy of our GSSA results assured, we turn to Table 5 to determine the ability of the HM calibration to resolve the Shimer puzzle in an RBC search model with curvature in preferences. Just as in Table 3, the top panel of Table 5 presents the empirical targets that need to be met in order to implement the HM calibration. A quick inspection of this table indicates that when the specifications with RBC preferences are calibrated to match the empirical targets suggested by HM, they do not generate sufficient labor market volatility.[15] While RBC researchers use somewhat different values of σ and ξ, results in Table 5 show that the failure of the HM calibration to resolve the Shimer puzzle in an RBC model, with standard preferences exhibiting risk aversion and finite Frisch elasticity, is a fairly robust outcome. In the most empirically relevant case with $\sigma = 2$ and $\xi = 3$, the volatilities of vacancies, unemployment, and tightness fall to less than half (50%, 49%, and 48%) of that in data. While a decrease in σ to 1 improves the results (76%, 73%, and 72%), a decrease in ξ to 2 makes things worse (46%, 45%, and 44%).

Therefore, the HM calibration fails to resolve the Shimer puzzle under a standard preference specification for an RBC model with risk-averse agents and finite Frisch elasticity. The results presented in this section indicate that the success of the HM calibration in resolving the Shimer puzzle in the standard DMP model was driven, in part, by the underlying assumption of linear preferences embedded in the model. Our sensitivity analysis shows that the failure of the HM calibration is not a result of a specific set of values a researcher may prefer. Rather, it is an outcome of the fact that the point estimates for which the HM calibration works is quite far outside the range of values considered plausible and used in the RBC literature to match business cycle moments of various macro variables.

Some insight into the reasons for this failure of the HM calibration can be obtained by combining its low implied value of workers' bargaining power[16] (see Table 2) with the wage equation, (19) to conclude that $w \approx -U_n/U_c$. Thus, just as in the case of a traditional RBC model, workers in our model, with labor search frictions and under the HM calibration, are on their iso-utility curves in the consumption-labor space (which is the same as their labor supply curve), albeit in an approximate sense. In particular, for our utility function, we have $w \approx dc/dn = \eta c^\sigma n^{1/\xi}$ and increasing σ or lowering ξ both make iso-utility curves

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[15]It should be noted that the full HM calibration cannot be implemented when $\sigma = 2$. Specifically, we still find that wage elasticity exceeds the empirical target after setting worker's bargaining power to its minimum value of 0. While this implies that the full calibration strategy is not used when $\sigma = 2$, the wage elasticity generated by the model is still close to the empirical target. As intuition provided later suggests, a further reduction in the elasticity of wages with respect to ALP in accordance with the HM calibration would further reduce labor market volatility.

[16]In HM, workers' bargaining power is approximately 0.05.

more curved in the consumption-labor space. Therefore, a given volatility of wages over the business cycle generates smaller volatility in c or n, when σ is larger or ξ is smaller.[17] As the HM calibration keeps the volatility of wages the same across calibrations, it makes the ceteris paribus assessment in the previous sentence the actual outcome. Specifically, the calibration targets the elasticity of wages with respect to ALP (0.449) and volatility of ALP (1.3 percent), thereby indirectly fixing the volatility of wages across our specifications.

4.3 Accuracy and Implications of Linear Approximation

Currently, there is an open debate regarding the impact of a solution's accuracy on the implied second moments in a model with DMP search frictions. Lan (2017) approximates the solution to an RBC search model under the standard labor market calibration (as in Shimer, 2005) with a Cobb-Douglas matching function, using both projection and perturbation methods. Lan finds that while the projection method is more accurate in terms of Euler residuals and DM-statistics, the second moments are not significantly different. However, Petrosky-Nadeau and Zhang (2016) solve the standard DMP search model under the HM calibration using similar methods and find that different methods yield significantly different results. Specifically, they find that once the model is solved more accurately, the HM calibration fails to resolve the Shimer puzzle as it yields an unemployment series that is almost twice as volatile as that found in the data. As the problem considered in this paper falls between these two papers—an RBC search model that implements the HM calibration—our results can potentially shed some light as to why these two papers reach very different conclusions.

Towards this end, in this section, we systematically explore the accuracy of linear approximation and its implications across our various specifications. Specifically, we first solve for the linearized approximation that implements the HM calibration and then assess the accuracy of our approximation by computing Euler residuals and performing a DM-statistics test on the predictability of ex-post errors, just as we did for as in the case of the GSSA solution. The results of these tests are presented in Table 6. Comparison of results in the first panel of Table 6 with the corresponding panel of Table 4 (for GSSA) shows that, with linearization, the root-mean-squared error (RMS) and the maximum-absolute error (∞-norm) for both Euler equations, (9) and (16), increase by at least an order of magnitude across all utility specifications. The second panel of Table 6 shows that the linearized solution

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[17]Note that c and n move together as a result of the interrelationship through the goods market clearing condition.

spectacularly fails the joint DM-statistics test, with a majority of the DM-statistics falling in the upper tail. Recall, the GSSA solutions passed the joint test, with approximately 5% of DM-statistics falling in the appropriate tails. The increased precision found using GSSA relative to the linear approximation can also be seen in Figures 1 and 2 which present time paths and histograms for the model's normalized Euler equation residuals under the RBC specification with $\sigma = 2$ and $\xi = 3$ (see online appendix for additional details).

4.3.1 Does the Inaccuracy of the Linearized Solution Matter?

Given the magnitude of the Euler residuals and the results of the DM-statistics test, it is clear that the solution based on our linear approximation is not accurate. But how does this inaccuracy affect our ability to test whether the HM calibration can resolve the Shimer puzzle? Table 7 summaries the results from the linearized solution. Comparing Table 7 with Table 5 indicates that our original conclusion remains. Namely, that while the HM calibration successfully resolves the Shimer puzzle in an RBC search model with linear preferences, it is unable to resolve the puzzle in a model with preferences more standard in the macro literature (e.g., risk averse agents and finite Frisch elasticity). Furthermore, our results indicate that our model's second moments are actually slightly larger when the *less-accurate* linear approximation is used.[18] This is in stark contrast to Petrosky-Nadeau and Zhang (2016) who reach an opposite conclusion: finding that their *more-accurate* solution significantly increases labor market volatility (unemployment is almost twice as volatile) under the HM calibration. Therefore, our results bolster the claim made by Lan (2017), suggesting that the accuracy of approximation, while important for statistical fit, has little impact on the second moments of the model.[19]

5 Conclusions

This paper applies the calibration strategy proposed by Hagedorn and Manovskii (2008) (HM) to an RBC search model similar to that proposed by Merz (1995) and Andolfatto (1996). We find that while the HM calibration significantly amplifies the volatility of vacan-

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[18]This is also reflected in the results presented in Table 3.

[19]Lan (2017) suggests that the difference with Petrosky-Nadau and Zhang (2016) is due to the choice of matching functions, whereby Lan allows matches to exceed vacancies in some periods. However, our results do not support this conjecture, as we adopt the same matching function as Petrosky-Nadau and Zhang (2016), yet our results favor Lan (2017). The principal difference with our model and the model presented in Petrosky-Nadeau and Zhang (2016) is the addition of capital in the production function.

cies, unemployment, and labor market tightness, relative to the standard calibration, it is not sufficient to resolve the Shimer Puzzle in our benchmark RBC specification. We conduct sensitivity analysis on the parameters of our utility function and find that the volatility of labor market variables falls as the coefficient of relative risk aversion rises. Ultimately, we find that the HM calibration is sufficient to resolve the Shimer puzzle only when one assumes that agents are risk neutral ($\sigma = 0$) and have infinite Frisch elasticity of labor supply. In addition, relaxing the strict assumption of an infinite Frisch elasticity only serves to deepen the Shimer puzzle, as volatility in the labor market is reduced further. We also examine the importance of accuracy of approximation in identifying the relative volatility of labor market variables by comparing results from a linear approximation to those from a non-linear approximation (GSSA). We find that the former dramatically fails a standard accuracy test based on DM-statistics, whereas the latter passes them. However, both approximation methods produce second moments for labor market variables that lead to similar conclusion about the ability of the HM calibration to resolve the Shimer puzzle.

The results of the paper demonstrate the inability of HM's calibration strategy to resolve the Shimer Puzzle for a typical calibration of the RBC search model. While the Frisch elasticity matters, the principal contributor to this failure is the degree of relative risk aversion normally present in RBC models. Specifically, we find that only if one is willing to assume risk-neutrality will the HM calibration come close to resolving the Shimer Puzzle in the RBC search model. We conclude from our results that if one is interested in matching the second moments of labor market variables within an RBC search model with risk-averse agents, additional features or amplification mechanisms (such as financial frictions) would need to be considered.

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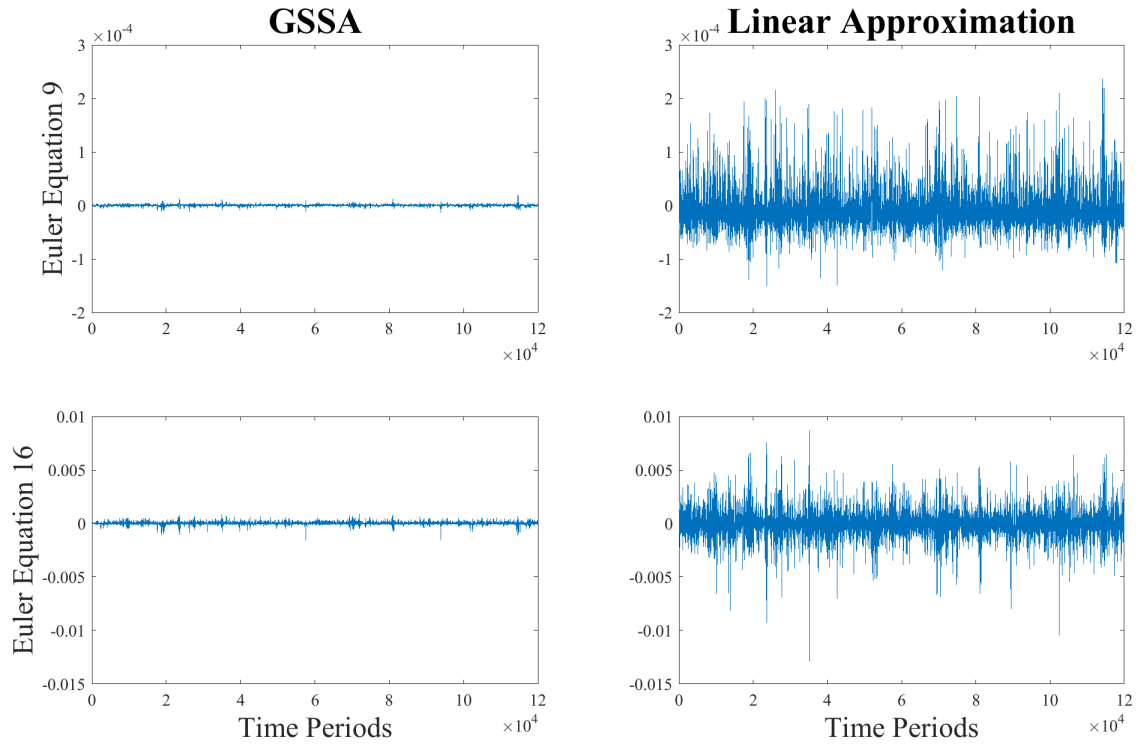# Figure 1: Euler Residuals: Time Paths ($\sigma = 2$ and $\xi = 3$)

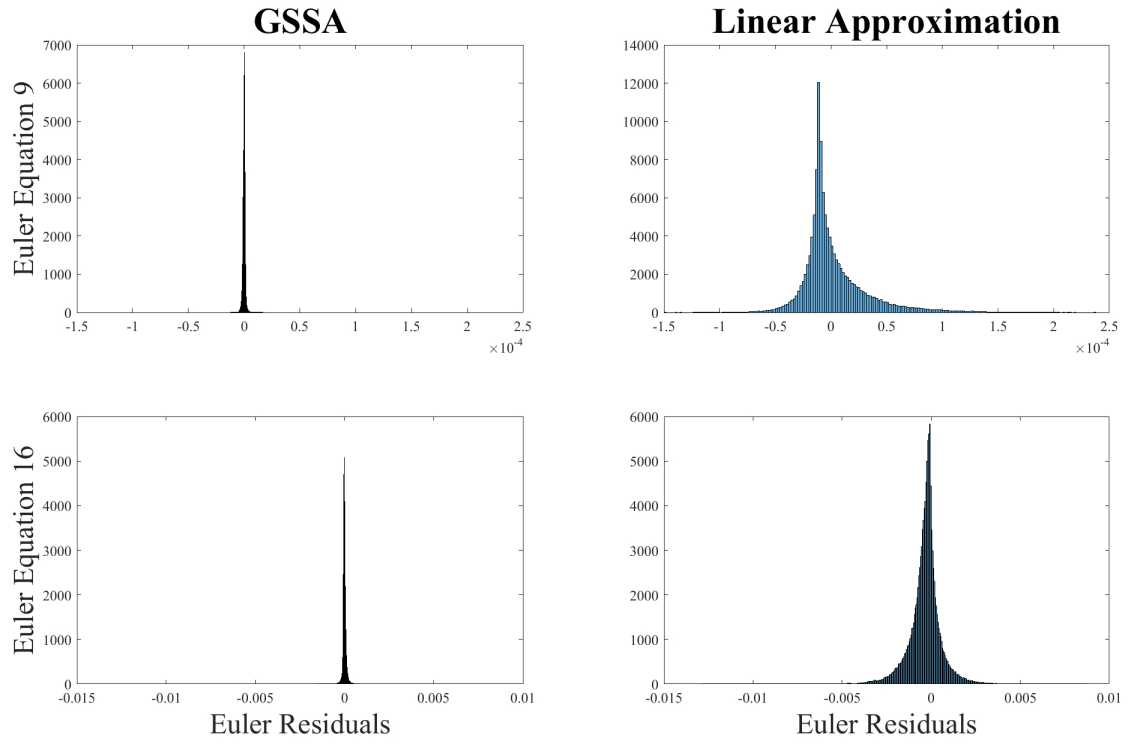# Figure 2: Euler Residuals: Histogram ($\sigma = 2$ and $\xi = 3$)

Table 1: Non-Adjusted Parameter Values

Parameters	α	β	δ	x	g
Values[a]	0.3333	0.9992	0.0021	0.0081	5.3794

[a] The parameter values listed above are defined as follows: $\alpha \equiv$ capital's share of output; $\beta \equiv$ subjective discount factor; $\delta \equiv$ depreciation rate of capital; $x \equiv$ exogenous separation rate; $g \equiv$ vacancy posting cost.

Table 2: Adjusted Parameter Values

	Linear	RBC					
	$\sigma = 0$	$\sigma = 1$			$\sigma = 2$		
Parameters[a]	$\xi = \infty$	$\xi = \infty$	$\xi = 3$	$\xi = 2$	$\xi = \infty$	$\xi = 3$	$\xi = 2$
			GSSA				
ρ_θ	0.9895	0.9916	0.9910	0.9909	0.9899	0.9898	0.9896
σ_ϵ	0.0022	0.0037	0.0037	0.0037	0.0036	0.0036	0.0035
b	0.0475	0.0213	0.0073	0.0005	0.0000	0.0000	0.0000
γ	0.4040	0.4035	0.4030	0.4025	0.4050	0.4025	0.4025
η	6.7215	0.9102	0.9347	0.9472	0.1233	0.1255	0.1266
			Linear Approximation				
ρ_θ	0.9895	0.9915	0.9911	0.9908	0.9899	0.9898	0.9896
σ_ϵ	0.0022	0.0037	0.0037	0.0036	0.0036	0.0036	0.0035
b	0.0312	0.0200	0.0073	0.0009	0.0000	0.0000	0.0000
γ	0.4125	0.4048	0.4025	0.4035	0.4085	0.4050	0.4025
η	6.8235	0.9106	0.9342	0.9458	0.1227	0.1250	0.1263

[a] The parameter values listed above are defined as follows: $\rho_\theta \equiv$ persistence of TFP; $\sigma_\epsilon \equiv$ volatility of innovation in TFP; $b \equiv$ worker bargaining power; $\gamma \equiv$ matching function parameter; $\eta \equiv$ weight on labor in utility function.

Table 3: Simulation Results for Linear Preferences

	Data	HM	GSSA	Linear
	Calibration Targets			
Wage Elasticity	0.449	0.449	0.449	0.450
Mean f	0.139	0.139	0.138	0.138
Mean θ	0.634	0.634	0.633	0.635
std(ALP)	1.300	1.300	1.300	1.300
Autocorr ALP	0.765	0.765	0.765	0.765
	Second Moments			
std(v)	13.900	16.900	15.008	20.161
std(u)	12.500	14.500	12.423	17.484
std(Φ)	25.900	29.200	25.299	34.351
corr(v,u)	-0.977	-0.724	-0.703	-0.658

Table 4: Accuracy Check for Linear and RBC Preferences: GSSA Approximation

| | Linear[a] | RBC | | | | | |
| | $\sigma = 0$ | $\sigma = 1$ | | | $\sigma = 2$ | | |
	$\xi = \infty$	$\xi = \infty$	$\xi = 3$	$\xi = 2$	$\xi = \infty$	$\xi = 3$	$\xi = 2$
Accuracy Measure				Euler Residuals			
Eq. 9 (RMS)	-	3.78e-8	3.05e-8	2.62e-8	3.62e-7	1.70e-7	1.16e-7
Eq. 9 (∞-norm)	-	2.20e-6	1.65e-5	1.86e-6	7.04e-6	2.39e-6	1.56e-5
Eq. 16 (RMS)	4.79e-6	6.87e-6	8.72e-6	9.60e-6	3.21e-5	1.80e-5	1.37e-5
Eq. 16 (∞-norm)	1.40e-3	4.07e-4	4.18e-4	4.59e-4	3.17e-4	2.82e-4	2.29e-4
				DM Statistics			
Eq. 9 (DM < Crit)	-	0.061	0.057	0.060	0.043	0.045	0.045
Eq. 9 (DM > Crit)	-	0.049	0.049	0.049	0.056	0.052	0.053
Eq. 16 (DM < Crit)	0.043	0.062	0.060	0.059	0.056	0.049	0.048
Eq. 16 (DM > Crit)	0.055	0.051	0.052	0.050	0.063	0.057	0.056
Joint (DM < Crit)	-	0.051	0.047	0.048	0.044	0.048	0.048
Joint (DM > Crit)	-	0.076	0.067	0.064	0.075	0.077	0.074

[a] Under linear preferences, equation (9) can be solved analytically using the log-normal property of $\log \theta$. Therefore, in this case we only present Euler residuals for equation (16).

Table 5: Simulation Results for Linear and RBC Preferences: GSSA Approximation

| | | Linear | RBC | | | | | |
| | | $\sigma = 0$ | | $\sigma = 1$ | | | $\sigma = 2$ | | |
	Data	$\xi = \infty$	$\xi = \infty$	$\xi = 3$	$\xi = 2$	$\xi = \infty$	$\xi = 3$	$\xi = 2$
Calibration Targets								
Wage Elasticity	0.449	0.449	0.450	0.447	0.448	0.465	0.529	0.552
Mean f	0.139	0.138	0.139	0.139	0.139	0.138	0.138	0.138
Mean θ	0.634	0.633	0.634	0.633	0.633	0.635	0.635	0.632
std(ALP)	1.300	1.300	1.298	1.297	1.298	1.298	1.300	1.298
Autocorr ALP	0.765	0.765	0.764	0.764	0.764	0.764	0.765	0.765
Second Moments								
std(v)	13.900	15.008	11.583	10.615	10.236	8.704	6.988	6.460
std(u)	12.500	12.423	9.967	9.089	8.738	7.670	6.088	5.609
std(Φ)	25.900	25.299	20.235	18.537	17.858	15.347	12.311	11.373
corr(v,u)	-0.977	-0.703	-0.763	-0.769	-0.771	-0.757	-0.772	-0.775

Table 6: Accuracy Check for Linear and RBC Preferences: Linear Approximation

| | Linear[a] | RBC | | | | | |
| | $\sigma = 0$ | $\sigma = 1$ | | | $\sigma = 2$ | | |
	$\xi = \infty$	$\xi = \infty$	$\xi = 3$	$\xi = 2$	$\xi = \infty$	$\xi = 3$	$\xi = 2$
Accuracy Measure				Euler Residuals			
Eq. 9 (RMS)	-	2.97e-6	2.75e-6	2.67e-6	4.46e-6	3.91e-6	3.73e-6
Eq. 9 (∞-norm)	-	2.37e-5	2.16e-5	2.07e-5	3.34e-5	3.02e-5	2.92e-5
Eq. 16 (RMS)	8.98e-4	2.82e-4	2.22e-4	2.02e-4	2.96e-4	1.86e-4	1.56e-4
Eq. 16 (∞-norm)	4.10e-3	2.96e-2	2.90e-3	2.50e-3	6.20e-3	3.00e-3	2.20e-3
				DM Statistics			
Eq. 9 (DM < Crit)	-	0.058	0.056	0.056	0.045	0.049	0.049
Eq. 9 (DM > Crit)	-	0.077	0.076	0.076	0.088	0.088	0.089
Eq. 16 (DM < Crit)	0.006	0.042	0.052	0.049	0.026	0.028	0.0341
Eq. 16 (DM > Crit)	0.483	0.135	0.099	0.089	0.190	0.176	0.161
Joint (DM < Crit)	-	0.000	0.000	0.000	0.000	0.000	0.000
Joint (DM > Crit)	-	0.998	0.856	0.594	1.000	1.000	1.000

[a] Under linear preferences, equation (9) can be solved analytically using the log-normal property of $\log \theta$. Therefore, in this case we only present Euler residuals for equation (16).

Table 7: Simulation Results for Linear and RBC Preferences: Linear Approximation

| | | Linear | RBC | | | | | |
| | | $\sigma = 0$ | $\sigma = 1$ | | | $\sigma = 2$ | | |
	Data	$\xi = \infty$	$\xi = \infty$	$\xi = 3$	$\xi = 2$	$\xi = \infty$	$\xi = 3$	$\xi = 2$
			Calibration Targets					
Wage Elasticity	0.449	0.450	0.449	0.449	0.449	0.450	0.523	0.548
Mean f	0.139	0.138	0.139	0.139	0.139	0.140	0.139	0.139
Mean θ	0.634	0.635	0.634	0.634	0.634	0.634	0.634	0.634
std(ALP)	1.300	1.300	1.300	1.299	1.300	1.299	1.299	1.300
Autocorr ALP	0.765	0.765	0.765	0.765	0.765	0.765	0.765	0.765
			Second Moments					
std(v)	13.900	20.161	12.222	11.069	10.644	9.495	7.477	6.786
std(u)	12.500	17.484	10.628	9.543	9.191	8.873	6.770	6.064
std(Φ)	25.900	34.351	21.395	19.323	18.623	17.024	13.315	12.035
corr(v,u)	-0.977	-0.658	-0.751	-0.755	-0.757	-0.716	-0.746	-0.753